Calculating mean power in distorted mains currents

The first part in this two-part series explained how high harmonic waveforms complicate power factor calculations and measurement. This part explains how a distorted current waveform transforms in shape through the mains reticulation network. The current waveform at the ballast may well be of a pulse-like nature and be in-phase with the voltage but ESKOM would never notice because the current waveform at the power station is a beautiful sinusoid *leading* the voltage. But how exactly can an in-phase current pulse affect the power factor? The answer lies in the mathematical analysis of the current waveforms.

Firstly, an understanding of a typical power reticulation network is necessary. The network is optimised to transmit currents at 50 Hz and so high frequencies attenuated – it is essentially a low pass filter. Higher frequencies are filtered by distributed line capacitance, hysteresis iron losses in the transformers, cable capacitance, skin effect, *etc*. An intuitive understanding of a pulsed current load may not be readily grasped and so, by using the principle of reciprocity, let us assume that a pulsed current is driven into the network from the ballast. Fourier analysis reveals that the current pulse, with infinite slope sides, is the summation of the fundamental (at 50 Hz) and harmonics out to infinity. In this case, by virtue of the network being a huge low pass filter, the current waveform at the power station will only consist of the fundamental, *lagging* the voltage, the higher harmonics being lost along the way.

Two mathematical treatments will reveal all; a piecewise linear approximation reveals the average power from a pulse current and Fourier analysis reveals that the power is all in the fundamental.

The typical current pulse in an off-the-line full wave rectified power supply is shown in Figure 1. The current pulse is narrow and almost in phase with the voltage.

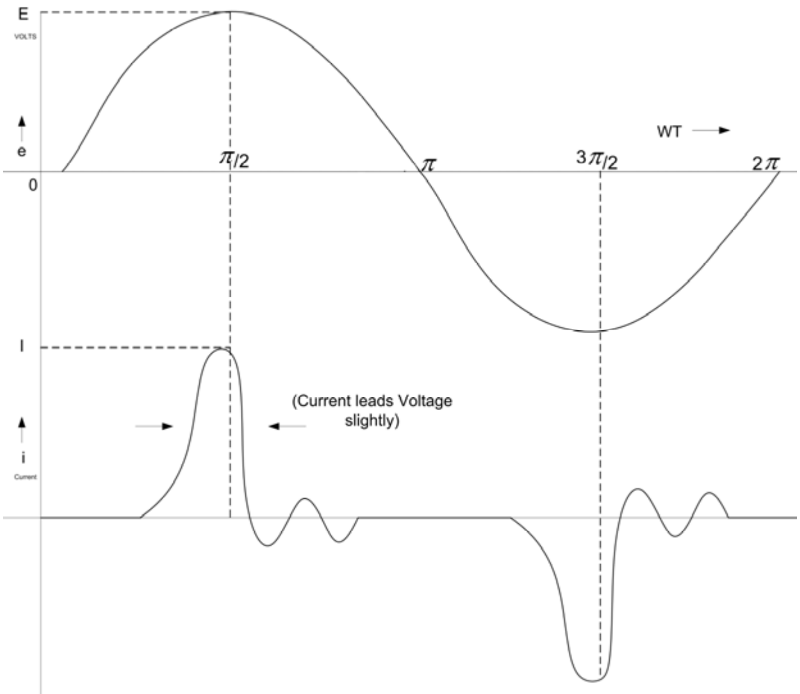

Figure 1

Our piecewise approximation of the current pulse is shown in Figure 2, its height being I and its width θ. Mean instantaneous power calculation is simply:

$$p = e.i$$

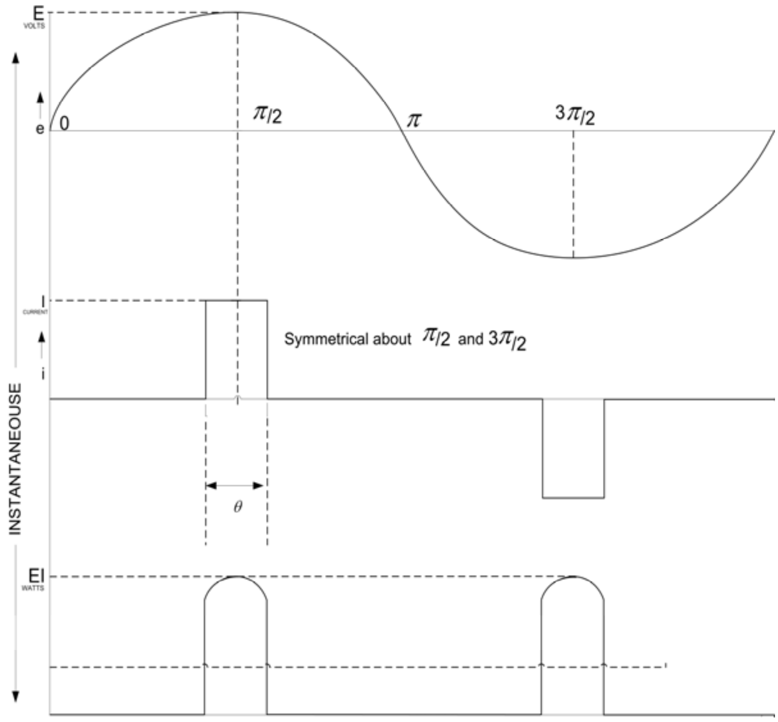

Figure 2

The mean power for a half cycle will be the instantaneous power averaged over a half cycle:

$$Mean\ power = \frac{1}{\pi}\int_{(\pi/2-\theta/2)}^{(\pi/2+\theta/2)} ei\ d(\omega t)$$

$$= \frac{EI}{\pi}\left[-\cos\omega t\right]_{(\pi/2-\theta/2)}^{(\pi/2+\theta/2)}$$

$$\therefore Mean\ power = \frac{2EI}{\pi}\sin\left(\frac{\theta}{2}\right)\ Watts$$

Note for small *θ*,

$$Mean\ power = \frac{EI\theta}{\pi} Watts, \qquad (1)$$

which is intuitively correct.

Calculating power from the fundamental current using Fourier analysis to isolate the fundamental current amplitude (dotted line in Figure 3) yields:

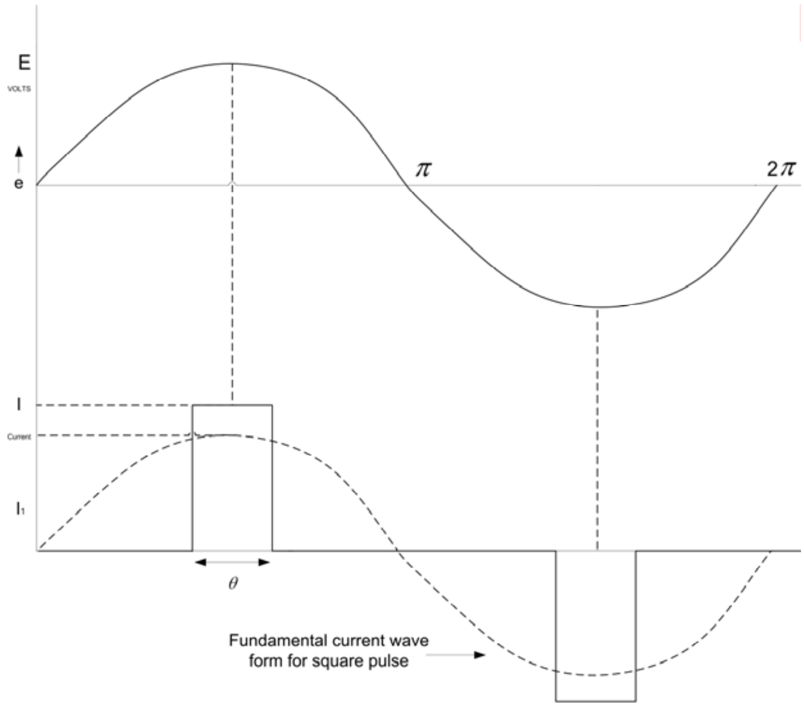

Figure 3

$$I_1 = \frac{I}{\pi}\left[\int_{\left(\pi/2-\theta/2\right)}^{\left(\pi/2+\theta/2\right)} \sin(\omega t)d\omega t + \int_{\left(3\pi/2-\theta/2\right)}^{\left(3\pi/2+\theta/2\right)} -\sin(\omega t)d\omega t\right]$$

This reduces to:

$$I_1 = \frac{4I}{\pi}\sin\left(\frac{\theta}{2}\right)$$

The power in the fundamental frequency, P_2, is given by:

$$P_1 = \frac{E}{\sqrt{2}} \times \frac{I_1}{\sqrt{2}} = \frac{E}{\sqrt{2}} \times \frac{4I \sin\left(\frac{\theta}{2}\right)}{\pi\sqrt{2}}$$

$$\therefore P_1 = \frac{2EI}{\pi} \sin\left(\frac{\theta}{2}\right) Watts$$

This is the same as in equation (1)! It proves that all of the power is from the current fundamental, all harmonics are quadrature or imaginary components. The current contribution from harmonics is significant, with pulsed currents, and explains why leading power factors of up to 0.5 may result.

This is the reason why.